Zadanie 1

TEMAT: Obliczyć $\lim\limits_{(x,y)\to(0,0)} f(x,y)$, jeżeli

a) $f(x,y)=\dfrac{1-\cos(x^2+y^2)}{(x^2+y^2)x^2y^2}$,

b) $f(x,y)=(\cos x^2y)^{\frac{1}{x^2+y^2}}$.

ROZWIĄZANIE:

a)

$$\lim_{(x,y)\to(0,0)}\frac{1-\cos(x^2+y^2)}{(x^2+y^2)x^2y^2}\overset{1-\cos 2x=2\sin^2 x}{=}\lim_{(x,y)\to(0,0)}\frac{2\left(\sin\frac{x^2+y^2}{2}\right)^2}{(x^2+y^2)x^2y^2}=$$

$$\lim_{(x,y)\to(0,0)}\frac{2\left(\sin\frac{x^2+y^2}{2}\right)^2}{4\left(\frac{x^2+y^2}{2}\right)^2}\frac{x^2+y^2}{x^2y^2}=\lim_{(x,y)\to(0,0)}\underbrace{\frac{1}{2}\Bigg(\underbrace{\frac{\sin\frac{x^2+y^2}{2}}{\frac{x^2+y^2}{2}}}_{\to 1}\Bigg)^2}_{\to\frac{1}{2}}\underbrace{\frac{x^2+y^2}{x^2y^2}}_{\to ?}$$

$$\lim_{(x,y)\to(0,0)}\frac{x^2+y^2}{x^2y^2}=\lim_{\substack{r\to0\\ \varphi-dmz}}\frac{r^2}{r^4\cos^2\varphi\sin^2\varphi}=\underbrace{\lim_{\substack{r\to0\\ \varphi-dmz}}\frac{1}{r^2\cos^2\varphi\sin^2\varphi}}_{nie\quad istnieje}\Rightarrow$$

$$\lim_{(x,y)\to(0,0)}\frac{1}{2}\left(\frac{\sin\frac{x^2+y^2}{2}}{\frac{x^2+y^2}{2}}\right)^2\frac{x^2+y^2}{x^2y^2}$$ <u>nie istnieje</u>

b)

$$\lim_{(x,y)\to(0,0)}(\cos x^2y)^{\frac{1}{x^2+y^2}}\overset{\cos 2x=1-2\sin^2 x}{=}\lim_{(x,y)\to(0,0)}\left(1-2\sin^2\frac{x^2y}{2}\right)^{\frac{1}{x^2+y^2}}=$$

$$\lim_{(x,y)\to(0,0)}\left(\left(1-2\sin^2\frac{x^2y}{2}\right)^{\frac{1}{-2\sin^2\frac{x^2y}{2}}}\right)^{\frac{-2\sin^2\frac{x^2y}{2}}{x^2+y^2}}=\lim_{(x,y)\to(0,0)}\underbrace{\left(\left(1-2\sin^2\frac{x^2y}{2}\right)^{\frac{1}{-2\sin^2\frac{x^2y}{2}}}\right)}_{(1+g(x))^{\frac{1}{g(x)}}\to e\ \longrightarrow e}^{\underbrace{\left(\frac{\sin\frac{x^2y}{2}}{\frac{x^2y}{2}}\right)^2}_{\to1}\underbrace{\frac{-2\left(\frac{x^2y}{2}\right)^2}{x^2+y^2}}_{\to ?}}$$

$$\lim_{(x,y)\to(0,0)} \frac{-2\left(\frac{x^2 y}{2}\right)^2}{x^2+y^2} = \lim_{(x,y)\to(0,0)} -\frac{x^4 y^2}{(x^2+y^2)} = \lim_{\substack{r\to 0\\ \varphi-dmz}} \frac{r^6 \cos^4\varphi \sin^2\varphi}{r^2} = \lim_{\substack{r\to 0\\ \varphi-dmz}} \underbrace{r^4}_{\to 0} \underbrace{\cos^4\varphi \sin^2\varphi}_{ograniczone} = 0 \Rightarrow$$

$$\lim_{(x,y)\to(0,0)} \underbrace{\left(\left(1-2\sin^2\frac{x^2 y}{2}\right)^{\frac{1}{-2\sin^2\frac{x^2 y}{2}}}\right)}_{\xrightarrow{(1+g(x))^{\frac{1}{g(x)}}\to e} e}^{\underbrace{\left(\frac{\sin\frac{x^2 y}{2}}{\frac{x^2 y}{2}}\right)^2}_{\to 1} \underbrace{\frac{-2\left(\frac{x^2 y}{2}\right)^2}{x^2+y^2}}_{\to ?}}$$

$$= e^{1\bullet 0} = \underline{\underline{1}}$$